Pasture Land Estimation

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Joint Work
with
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Introduction
The 2015 Pasture Land Study frame includes all segments from the 2015 Rotation Sample that were observed in 2008.

- The frame excludes points from Alaska, Hawaii, and the Caribbean.
- Segments must have at least one eligible point.
- Segments containing points in an on-site survey sample in 2003 or later are ineligible for sample selection.
NRI Grazing Land on-site data collection began in 2003.

Nineteen states were included in the sampling: AZ, CA, CO, FL, ID, KS, LA, MT, NE, NV, NM, ND, OK, OR, SD, TX, UT, WA, and WY.

Data were also collected in HI, but are too sparse to be included in summary reports.

Site revisits started in 2012 where on-site data were collected 5 and 8 years earlier.

Generally 700 segments from each of the earlier years were selected.
Clustering of counties within each state were purposively selected to decrease the cost of travel to sample sites.

Samples were not selected from the same counties two years in a row.

The counties included were those with the largest rangeland acreage; some counties with few rangeland acres were never included in the samples.

Segments with at least two non-Federal rangeland points were eligible for sample selection.

Beginning with 2010, at least one segment with only one non-Federal rangeland point was selected for each county.

Segments within counties were selected as a simple random sample from the list segments eligible for selection.
# Yearly Sampling Pattern

## The Overview of the Sampling Pattern for the NRI Range Study

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<td>X(562)</td>
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Goals

- Estimate the levels for the period 2004-2010.
- Estimate the levels for the period 2011-2015.
Estimates
Define four sets of panels as follows:

- $P_1$: panel containing points that are observed in both periods with 8 years gap.
- $P_2$: panel containing points that are observed in both periods with 5 years gap.
- $P_3$: panel containing points that are observed only in period I.
- $P_4$: panel containing points that are observed only in period II.
Panel Definitions

Using these panels, we can create two different estimates each for $\theta_I$ and $\theta_{II}$.

Define

- $\theta_I$: population total of variable $y$ in period I.
- $\theta_{II}$: population total of variable $y$ in period II.

We can construct weights for the groups $(P_1 + P_3, P_2 + P_3, P_1 + P_4, P_2 + P_4)$.

- Note that 2010 point gen data is used for $P_1 + P_3$ and $P_2 + P_3$.
- The 2012 point gen data is used for $P_1 + P_4$ and $P_2 + P_4$.

The resulting estimates for each of these panel combinations are called $\hat{\theta}_{I,13}, \hat{\theta}_{I,23}, \hat{\theta}_{II,14},$ and $\hat{\theta}_{II,24}$, where the first two are estimating $\theta_I$ and the last two are estimating $\theta_{II}$.
Period I (2004 − 2010)

\[ P_1 + P_3 \Rightarrow \hat{\theta}_{I,13} \]
\[ P_2 + P_3 \Rightarrow \hat{\theta}_{I,23} \]

Period II (2011 − 2015)

\[ P_1 + P_4 \Rightarrow \hat{\theta}_{II,14} \]
\[ P_2 + P_4 \Rightarrow \hat{\theta}_{II,24} \]

\[ \theta_I \]
\[ \theta_{II} \]
Using the GLS estimator,

\[
\begin{pmatrix}
\hat{\theta}_{I,13} \\
\hat{\theta}_{I,23} \\
\hat{\theta}_{II,14} \\
\hat{\theta}_{II,24}
\end{pmatrix}
= \begin{pmatrix}
1 & 0 \\
1 & 0 \\
0 & 1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\theta_I \\
\theta_{II}
\end{pmatrix}
+ \varepsilon
\] (2)

where \(\varepsilon \sim (0, V)\).

Define \(\hat{y} = \begin{pmatrix}
\hat{\theta}_{I,13} \\
\hat{\theta}_{I,23} \\
\hat{\theta}_{II,14} \\
\hat{\theta}_{II,24}
\end{pmatrix}\), \(X = \begin{pmatrix}
1 & 0 \\
1 & 0 \\
0 & 1 \\
0 & 1
\end{pmatrix}\), and \(\theta = \begin{pmatrix}
\theta_I \\
\theta_{II}
\end{pmatrix}\).

Then

\[
\hat{\theta} = \begin{pmatrix}
\hat{\theta}_I \\
\hat{\theta}_{II}
\end{pmatrix}
= (X^T V^{-1} X)^{-1} X^T V^{-1} \hat{y}
\] (3)
In order to obtain the covariance matrix $V$, assume a first order autoregressive process AR(1) with a parameter $\phi$. Under this assumption, the covariances are defined as follows:

$$
cov\left(\hat{\theta}_I,_{13}, \hat{\theta}_{II},_{14}\right) \propto \frac{\phi^8 n_1 \hat{\sigma}_{13} \hat{\sigma}_{14}}{n_{13} n_{14}}
$$

$$
cov\left(\hat{\theta}_I,_{23}, \hat{\theta}_{II},_{24}\right) \propto \frac{\phi^5 n_2 \hat{\sigma}_{23} \hat{\sigma}_{24}}{n_{23} n_{24}}
$$

$$
cov\left(\hat{\theta}_I,_{13}, \hat{\theta}_I,_{23}\right) \propto \frac{\phi^0 n_3 \hat{\sigma}_{13} \hat{\sigma}_{23}}{n_{13} n_{23}}
$$

$$
cov\left(\hat{\theta}_{II},_{14}, \hat{\theta}_{II},_{24}\right) \propto \frac{\phi^0 n_4 \hat{\sigma}_{14} \hat{\sigma}_{24}}{n_{14} n_{24}}
$$

and all other covariances are equal to 0, assuming different panels are uncorrelated.
How to Obtain $V$

$n_i$ is the number of segments in panel $i$, and $n_{ij}$ is the number of segments in panels $P_i + P_j$ where $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$.

\begin{itemize}
  \item $\hat{\sigma}_{13} = \sqrt{n_{13} \hat{\text{Var}}(\hat{\theta}_{I,13})}$,
  \item $\hat{\sigma}_{14} = \sqrt{n_{14} \hat{\text{Var}}(\hat{\theta}_{II,14})}$,
  \item $\hat{\sigma}_{23} = \sqrt{n_{23} \hat{\text{Var}}(\hat{\theta}_{I,23})}$,
  \item $\hat{\sigma}_{24} = \sqrt{n_{24} \hat{\text{Var}}(\hat{\theta}_{II,24})}$.
\end{itemize}
A delete a group jackknife replication method is used to calculate the variance of estimates.

For NRI related surveys, this method involves defining 29 replicates, which lead to 29 replicate weights $\theta^{[b] \ I,13}$, $\theta^{[b] \ II,23}$, $\theta^{[b] \ I,14}$, and $\theta^{[b] \ II,24}$, $b = 1, 2, \ldots, 29$. 
Then the variance of $\hat{\theta}_{1,13}$, for example, is calculated as:

$$\hat{\text{Var}} \left( \hat{\theta}_{1,13} \right) = \frac{28}{29} \sum_{b=1}^{B} \left( \hat{\theta}_{[b]}_{1,13} - \hat{\theta}_{1,13} \right)^2,$$

where

$$\hat{\theta}_{1,13} = \frac{1}{29} \sum_{b=1}^{B} \hat{\theta}_{[b]}_{1,13}.$$

Other variances are calculated in the same manner.
How to Obtain $\phi$

$\phi$ needs to be estimated using the real data. Define

$$\hat{\phi}_1^8 = \frac{\sum_{i \in P_1} (y_{i,0} - \bar{y}_0)(y_{i,8} - \bar{y}_8)}{\sqrt{\sum_{i \in P_1} (y_{i,0} - \bar{y}_0)^2 \sum_{i \in P_1} (y_{i,8} - \bar{y}_8)^2}}$$  \hfill (6)$$

$$\hat{\phi}_2^5 = \frac{\sum_{i \in P_1} (y_{i,0} - \bar{y}_0)(y_{i,5} - \bar{y}_5)}{\sqrt{\sum_{i \in P_1} (y_{i,0} - \bar{y}_0)^2 \sum_{i \in P_1} (y_{i,5} - \bar{y}_5)^2}}$$  \hfill (7)$$

where $y_{i,0}$ is the observation at year $t$, $y_{i,5}$ is the observation at year $t + 5$, and $y_{i,8}$ is the observation at year $t + 8$. 
Then

$$\hat{\phi} = \alpha \hat{\phi}_1 + (1 - \alpha) \hat{\phi}_2$$  \hspace{1cm} (8)

where

$$\alpha = \frac{1}{\frac{1}{n_1} + \frac{1}{n_2}}$$  \hspace{1cm} (9)

with $n_1$ being the number of points in $P_1$ and $n_2$ being the number of points in $P_2$. 
Define 

\[ \hat{\theta} = \begin{pmatrix} \hat{\theta}_I \\ \hat{\theta}_{II} \end{pmatrix} \]

and its variance estimator,

\[ \hat{\text{Var}} \left( \hat{\theta} \right) = \left( X^T V^{-1} X \right)^{-1} \]

We can estimate the change between the two periods as:

Estimated Change \( = \hat{\theta}_{II} - \hat{\theta}_I \) \hfill (10)

Standard Error of Estimated Change \( = \sqrt{\left( \begin{array}{cc} 1 & -1 \\ \end{array} \right) \hat{\text{Var}} \left( \hat{\theta} \right) \left( \begin{array}{cc} 1 \\ -1 \end{array} \right)} \) \hfill (11)
Providing Weights and Replicate Weights for the Change Estimates
The change in rangeland estimate has been set up to work for a given $y$, i.e. the calculation of covariance matrix $V$ depends on the choice of $y$.

We want to develop a set up that works for any $y$.

To do this we will create weights and replicate weights to estimate $\hat{\theta}_{II} - \hat{\theta}_I$ and $SE(\hat{\theta}_{II} - \hat{\theta}_I)$.

The best $V$ matrix for the variable of interest would require a weight table for each variable of interest and knowing these $y$’s ahead of time.

Alternatively, we could set $V$ to be the identity matrix, which does not depend on knowing $y$’s, but it does lose efficiency.
As a compromise between these two methods, we can choose a set of important variables, $y_g$, $g = 1, \ldots, G$, here $G = 48$, and estimate the covariance matrix $V_g$ for each variable $y_g$ using the data.

Roni helped identify the important variables.

Then we take $\tilde{V} = \frac{1}{G} \sum_{g=1}^{G} V_g$, and use $\tilde{V}$ in

$$\hat{\theta} = \left( \begin{array}{c} \hat{\theta}_I \\ \hat{\theta}_{II} \end{array} \right) = (X^T \tilde{V}^{-1} X)^{-1} X^T \tilde{V}^{-1} \hat{y}$$

Then we obtain

$$\hat{\theta} = \left( \begin{array}{c} \hat{\theta}_I \\ \hat{\theta}_{II} \end{array} \right) = (X^T \tilde{V}^{-1} X)^{-1} X^T \tilde{V}^{-1} \hat{y}$$
To construct the weight table that will be used to obtain $\hat{\theta}_I$ and $\hat{\theta}_{II}$, define

$$(X^T \tilde{V}^{-1} X)^{-1} X^T \tilde{V}^{-1} \overset{\triangle}{=} A \overset{\triangle}{=} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}. \quad (13)$$

So

$$\hat{\theta} = \begin{pmatrix} \hat{\theta}_I \\ \hat{\theta}_{II} \end{pmatrix} = A \hat{y} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} \sum_{i \in P_1 + P_3} w_{13,i} y_i \\ \sum_{i \in P_2 + P_3} w_{23,i} y_i \\ \sum_{i \in P_1 + P_4} w_{14,i} y_i \\ \sum_{i \in P_2 + P_4} w_{24,i} y_i \end{bmatrix} \quad (14)$$
where

- $y_i$ is any variable of interest
- $w_{13,i}$ is the weight for point $i$ in the $P_1 + P_3$ data set,
- $w_{23,i}$ is the weight for point $i$ in the $P_2 + P_3$ data set,
- $w_{14,i}$ is the weight for point $i$ in the $P_1 + P_4$ data set,
- $w_{24,i}$ is the weight for point $i$ in the $P_2 + P_4$ data set.
\[ \hat{\theta} = \begin{pmatrix} \hat{\theta}_I \\ \hat{\theta}_{II} \end{pmatrix} = A\hat{y} = 
\begin{pmatrix}
a_{11} \sum_{i \in P_1 + P_3} w_{13, i} y_i + a_{12} \sum_{i \in P_2 + P_3} w_{23, i} y_i + a_{13} \sum_{i \in P_1 + P_4} w_{14, i} y_i + a_{14} \sum_{i \in P_2 + P_4} w_{24, i} y_i \\
a_{21} \sum_{i \in P_1 + P_3} w_{13, i} y_i + a_{22} \sum_{i \in P_2 + P_3} w_{23, i} y_i + a_{23} \sum_{i \in P_1 + P_4} w_{14, i} y_i + a_{24} \sum_{i \in P_2 + P_4} w_{24, i} y_i 
\end{pmatrix}.\]
\[ w_{i} = \begin{cases} 
   a_{11} w_{13,i} & \text{if } i \in P_1 \text{ and observed in 2004-2010} \\
   a_{12} w_{23,i} & \text{if } i \in P_2 \text{ and observed in 2004-2010} \\
   a_{11} w_{12,i} + a_{13} w_{23,i} & \text{if } i \in P_3 \\
   a_{13} w_{14,i} & \text{if } i \in P_1 \text{ and observed in 2011-2015} \\
   a_{14} w_{24,i} & \text{if } i \in P_2 \text{ and observed in 2011-2015} \\
   a_{13} w_{14,i} + a_{14} w_{24,i} & \text{if } i \in P_4
\]
\[ w_{IIi} = \begin{cases} 
  a_{21}w_{13,i} & \text{if } i \in P_1 \text{ and observed in 2004-2010} \\
  a_{22}w_{23,i} & \text{if } i \in P_2 \text{ and observed in 2004-2010} \\
  a_{21}w_{13,i} + a_{22}w_{23,i} & \text{if } i \in P_3 \\
  a_{23}w_{14,i} & \text{if } i \in P_1 \text{ and observed in 2011-2015} \\
  a_{24}w_{24,i} & \text{if } i \in P_2 \text{ and observed in 2011-2015} \\
  a_{23}w_{14,i} + a_{24}w_{24,i} & \text{if } i \in P_4 
\end{cases} \]
\[
\begin{aligned}
    w_{li}^{[b]} &= \begin{cases} 
    a_{11} w_{13,i}^{[b]} & \text{if } i \in P_1 \text{ and observed in 2004-2010} \\
    a_{12} w_{23,i}^{[b]} & \text{if } i \in P_2 \text{ and observed in 2004-2010} \\
    a_{11} w_{12,i}^{[b]} + a_{13} w_{23,i}^{[b]} & \text{if } i \in P_3 \\
    a_{13} w_{14,i}^{[b]} & \text{if } i \in P_1 \text{ and observed in 2011-2015} \\
    a_{14} w_{24,i}^{[b]} & \text{if } i \in P_2 \text{ and observed in 2011-2015} \\
    a_{13} w_{14,i}^{[b]} + a_{14} w_{24,i}^{[b]} & \text{if } i \in P_4 
    \end{cases}
\end{aligned}
\]
\[ w_{IIi}^{[b]} = \begin{cases} 
    a_{21}w_{13,i}^{[b]} & \text{if } i \in P_1 \text{ and observed in 2004-2010} \\
    a_{22}w_{23,i}^{[b]} & \text{if } i \in P_2 \text{ and observed in 2004-2010} \\
    a_{21}w_{13,i}^{[b]} + a_{22}w_{23,i}^{[b]} & \text{if } i \in P_3 \\
    a_{23}w_{14,i}^{[b]} & \text{if } i \in P_1 \text{ and observed in 2011-2015} \\
    a_{24}w_{24,i}^{[b]} & \text{if } i \in P_2 \text{ and observed in 2011-2015} \\
    a_{23}w_{14,i}^{[b]} + a_{24}w_{24,i}^{[b]} & \text{if } i \in P_4 
\end{cases} \]
### Weight Table

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</tr>
</tbody>
</table>

Table: Final weight Table
Now the point estimator can be calculated as

\[ \hat{\theta}_I = \sum_{i \in S} w_{li} y_i \]

\[ \hat{\theta}_{II} = \sum_{i \in S} w_{lII} y_i \]

with the change estimator being

\[ \hat{\theta}_{II} - \hat{\theta}_I \]

and the standard error for the change is:

\[ \hat{V}(\hat{\theta}_{II} - \hat{\theta}_I) = \hat{V}(\hat{\theta}_I) + \hat{V}(\hat{\theta}_{II}) - 2 \hat{Cov}(\hat{\theta}_I, \hat{\theta}_{II}) \]

\[ SE(\hat{\theta}_I, \hat{\theta}_{II}) = \sqrt{\hat{V}(\hat{\theta}_{II} - \hat{\theta}_I)} \]
The variance estimator can be calculated as

\[
\hat{V}(\hat{\theta}_I) = \frac{28}{29} \sum_{b=1}^{29} \left( \hat{\theta}^{[b]}_I - \hat{\theta}_I \right)^2 \\
\hat{V}(\hat{\theta}_{II}) = \frac{28}{29} \sum_{b=1}^{29} \left( \hat{\theta}^{[b]}_{II} - \hat{\theta}_{II} \right)^2 \\
\hat{\text{Cov}}(\hat{\theta}_I, \hat{\theta}_{II}) = \frac{28}{29} \sum_{b=1}^{29} \left( \hat{\theta}^{[b]}_I - \hat{\theta}_I \right) \left( \hat{\theta}^{[b]}_{II} - \hat{\theta}_{II} \right).
\]

where

\[
\hat{\theta}^{[b]}_I = \sum_{i \in S} w^{[b]}_{ii} y_i \\
\hat{\theta}^{[b]}_{II} = \sum_{i \in S} w^{[b]}_{iII} y_i
\]
Using the weight we obtained above and supposing that we are interested in a domain estimator, then define

\[
\tilde{\theta}_I = \frac{\sum_{i \in S} w_{II} \delta_i y_i}{\sum_{i \in S} w_{II} \delta_i}
\]

\[
\tilde{\theta}_{II} = \frac{\sum_{i \in S} w_{III} \delta_i y_i}{\sum_{i \in S} w_{III} \delta_i}
\]

where \( \delta_i \) is the indicator of whether point \( i \) lies in the particular domain for which the estimate is being constructed.
Similarly, the replicated estimators are defined as

\[
\tilde{\theta}^{[b]}_i = \frac{\sum_{i \in S} w_{li}^{[b]} \delta_i y_i}{\sum_{i \in S} w_{li}^{[b]} \delta_i}
\]

\[
\tilde{\theta}^{[b]}_{II} = \frac{\sum_{i \in S} w_{III}^{[b]} \delta_i y_i}{\sum_{i \in S} w_{III}^{[b]} \delta_i}
\]

where \( b = 1, \ldots, 29 \), \( S \) is the combined points for 2004-2010 and 2011-2015, and \( \delta_i \) is the indicator of whether point \( i \) is contained in the particular domain of interest for which the estimate is being constructed.
The estimated change is
\[ \hat{\theta}_{II} - \hat{\theta}_I, \]  
and the standard error for the change is:
\[
\hat{V}(\hat{\theta}_{II} - \hat{\theta}_I) = \hat{V}(\hat{\theta}_I) + \hat{V}(\hat{\theta}_{II}) - 2\hat{Cov}(\hat{\theta}_I, \hat{\theta}_{II})
\]
\[
SE(\hat{\theta}_I, \hat{\theta}_{II}) = \sqrt{\hat{V}(\hat{\theta}_{II} - \hat{\theta}_I)}
\]
The domain variance estimator can be calculated as

\[
\hat{V}(\bar{\hat{\theta}}_I) = \frac{28}{29} \sum_{b=1}^{29} (\bar{\hat{\theta}}_I^{[b]} - \bar{\hat{\theta}}_I)^2
\]

\[
\hat{V}(\bar{\hat{\theta}}_{II}) = \frac{28}{29} \sum_{b=1}^{29} (\bar{\hat{\theta}}_{II}^{[b]} - \bar{\hat{\theta}}_{II})^2
\]

\[
\hat{Cov}(\bar{\hat{\theta}}_I, \bar{\hat{\theta}}_{II}) = \frac{28}{29} \sum_{b=1}^{29} (\bar{\hat{\theta}}_I^{[b]} - \bar{\hat{\theta}}_I)(\bar{\hat{\theta}}_{II}^{[b]} - \bar{\hat{\theta}}_{II}).
\]
Results
Many of the variables estimated were indicator variables, which leads to the results being in percentages.

The value in the table indicates the percent increase or decrease in land for that particular variable.

The following table shows the period 1 estimates, period 2 estimates, and change estimates as well as the standard errors for each.
<table>
<thead>
<tr>
<th>State</th>
<th>Percent of Bare Ground</th>
<th></th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period I</td>
<td>Period II</td>
<td></td>
</tr>
<tr>
<td>Kansas</td>
<td>4.81 (.42)</td>
<td>4.36 (.76)</td>
<td>-0.45 (.82)</td>
</tr>
<tr>
<td>North Dakota</td>
<td>2.67 (.54)</td>
<td>1.99 (.63)</td>
<td>-0.68 (.74)</td>
</tr>
<tr>
<td>Washington</td>
<td>10.50 (1.59)</td>
<td>8.69 (2.17)</td>
<td>-1.80 (2.61)</td>
</tr>
<tr>
<td>National</td>
<td>14.44 (.32)</td>
<td>17.45 (1.23)</td>
<td>3.00 (1.16)</td>
</tr>
</tbody>
</table>
## Table of Results

<table>
<thead>
<tr>
<th>State</th>
<th>Percent of Non-native Plants Present</th>
<th>Period I</th>
<th>Period II</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kansas</td>
<td>79.44 (2.82)</td>
<td>75.74 (4.46)</td>
<td>-3.71 (4.77)</td>
<td></td>
</tr>
<tr>
<td>North Dakota</td>
<td>70.29 (3.42)</td>
<td>79.68 (3.60)</td>
<td>9.39 (4.53)</td>
<td></td>
</tr>
<tr>
<td>Washington</td>
<td>95.24 (6.95)</td>
<td>96.02 (3.82)</td>
<td>0.78 (4.95)</td>
<td></td>
</tr>
<tr>
<td>National</td>
<td>53.53 (1.24)</td>
<td>55.27 (1.67)</td>
<td>1.74 (2.26)</td>
<td></td>
</tr>
</tbody>
</table>
Questions?