An approximate Bayesian inference under informative sampling

Zhonglei Wang
(Joint work with Dr. Jae-Kwang Kim and Dr. Shu Yang)

Center for Survey Statistics and Methodology
Iowa State University

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Analyzing survey data to make inference about finite population parameters is of major interest in survey sampling.

Bayesian approaches are widely used to handle complex problems, but non-informative sampling design is often assumed.

Informative sampling design (Fuller, 2009, §6.3.2; Pfeffermann et al., 2006): the sampling probabilities are related to the values of the outcome variable after conditioning on the model covariates.

The goal is to develop a new Bayesian method under informative sampling settings such that

- it only requires the most straightforward data model,
- it is easy to implement in practice.
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Problem setup

- Finite population $Y_N = (y_1, \cdots, y_N)$ is a random sample of size $N$ from a superpopulation $\zeta$ with density $f(y; \theta)$ for some $\theta \in \Theta$.
- Let $Y_N = (Y_n, Y_{N-n})$, where $Y_n$ and $Y_{N-n}$ are the sampled and nonsampled parts.
- The parameter of interest is $Q_N(Y_N)$, such as $Q(Y_N) = N^{-1} \sum_{i=1}^{N} y_i$. 
Classical Bayesian inference procedures

Step 1. Generate $\theta$ from

$$\theta^* \sim p(\theta \mid Y_n) \propto \tilde{f}_1(Y_n; \theta)\pi(\theta),$$

where $\tilde{f}_1$ is the density of sample $Y_n$. Under non-informative sampling, $\tilde{f}_1(Y_n; \theta) = \prod_{i=1}^n f(y_i; \theta)$.

Step 2. Generate $y_i$ for $i = n + 1, \ldots, N$ from

$$y_i^* \sim f(y_i; \theta^*),$$

where $\theta^*$ is generated at Step 1.

Step 3. The posterior value of $Q_N$ given $Y_n$ is computed by

$$Q_N^* = Q\{(Y_n, Y_N^*-n)\},$$

where $Y_N^*-n = (y_{n+1}^*, \ldots, y_N^*)$. 
The sample density, \( \tilde{f}_1 \), cannot be computed directly in the informative sampling case.

We consider an alternative approach that does not use the sample-data likelihood \( \tilde{f}_1 \).

Let \( \hat{\theta} \) be a direct estimate of \( \theta \), which satisfies

\[
\hat{\theta} = \theta + o_p(1).
\]
To obtain a consistent estimator $\hat{\theta}$, one option is to solve the following pseudo score equation for $\theta$

$$\hat{S}(\theta) = \sum_{i=1}^{n} w_i S(\theta; y_i) = 0,$$

(3)

where $w_i$ is the sampling weight for element $i$, and $S(\theta; y) = \partial \log f(y; \theta)/\partial \theta$ is the score function of $\theta$. 
Proposed method (Cont’d)

- We propose to use

\[ \theta^* \sim p(\theta \mid \hat{\theta}) \propto g_1(\hat{\theta} \mid \theta)\pi(\theta), \] (4)

where \( g_1(\hat{\theta} \mid \theta) \) is the sampling distribution of \( \hat{\theta} \).

- Under some regularity conditions (Fuller, 2009, §1.3.2), it can be shown that

\[ \{ \hat{V}(\hat{\theta})\}^{-1/2} (\hat{\theta} - \theta) \mid \theta \to^d \mathcal{N}(0, I) \] (5)

as \( n \to \infty \), where \( \hat{V}(\hat{\theta}) \) is a consistent variance estimator of \( \hat{\theta} \), and \( I \) is the identity matrix.

- Taylor linearization is often involved.
One alternative

- We can also consider to generate $\theta^*$ by

$$
\theta^* \sim p\{\theta \mid \hat{S}(\theta)\} \propto g_2\{\hat{S}(\theta) \mid \theta\} \pi(\theta),
$$

where $g_2\{\hat{S}(\theta) \mid \theta\}$ is the sampling distribution of the pseudo score function $\hat{S}(\theta)$.

- By assuming asymptotic normality for $\hat{S}(\theta)$, we can use

$$
g_2\{\hat{S}(\theta) \mid \theta\} = \exp\left[-\frac{1}{2} \hat{S}(\theta)^T \{\hat{V}_{ss}(\theta)\}^{-1} \hat{S}(\theta) - \frac{1}{2} \log \left| \hat{V}_{ss}(\theta)/(2\pi) \right| \right],
$$

where $\hat{V}_{ss}(\theta)$ is a design-consistent estimator of the sampling variance of $\hat{S}(\theta)$.

- Adaptive rejection Metropolis sampling algorithm within Gibbs sampling (Gikls et al., 1995) can be used for implementation.
Once the posterior value of $\theta$, say $\theta^*$, are generated, we obtain $y_i^* \sim f(y_i; \theta^*)$ for $i = 1, \ldots, N$ and compute $Q_N^* = Q(Y_N^*)$ as the posterior value of $Q_N$, where $Y_N^* = (y_1^*, \ldots, y_N^*)$. 
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Semi-parametric method

Problem setup

- Suppose that the parameter of interest $\theta$ satisfies

$$\sum_{i=1}^{N} U(\theta; y_i) = 0,$$

where $U(\theta; y)$ is the estimating function for $\theta$.

- For example, $U(\theta; y) = I(y < 1) - \theta$ gives $\theta = N^{-1} \sum_{i=1}^{N} I(y_i < 1)$, where $I(y < 1) = 1$ if $y < 1$, and 0 otherwise.
A consistent estimator $\hat{\theta}$ solves

$$
\hat{U}(\theta) = \sum_{i=1}^{n} w_i U(\theta; y_i) = 0. \tag{6}
$$

Assume that the solution to (6) exists uniquely almost everywhere, and a design consistent estimator of $\text{var}\{\hat{U}(\theta)\}$ is available for fixed $\theta$.

If $w_i = 1/\pi_i$, then Horvitz-Thompson variance estimator (Horvitz & Thompson, 1952) is

$$
\hat{V}\{\hat{U}(\theta)\} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{(\pi_{ij} - \pi_i \pi_j)}{\pi_{ij}} \frac{1}{\pi_i} \frac{1}{\pi_j} U(\hat{\theta}; y_i) U(\hat{\theta}; y_j)^T, \tag{7}
$$

where $\pi_i$ and $\pi_{ij}$ are the first and second order inclusion probability.
Proposed semi-parametric method

- Under certain regular conditions, we have

\[
\hat{V}\{\hat{U}(\theta)\}^{-1/2}\hat{U}(\theta) \overset{d}{\rightarrow} N(0, I)
\]

as \( n \rightarrow \infty \).

- Thus, we can construct a pseudo-likelihood function of \( \theta \) as

\[
g\{\hat{U}(\theta) \mid \theta\} = \exp\left[-\frac{1}{2} \hat{U}(\theta)^T \hat{V}\{\hat{U}(\theta)\}^{-1}\hat{U}(\theta) - \frac{1}{2} \log \left| \hat{V}\{\hat{U}(\theta)\} / (2\pi) \right| \right]
\]

- The approximate posterior distribution of \( \theta \) becomes

\[
p\{\theta \mid \hat{U}(\theta)\} \propto g\{\hat{U}(\theta) \mid \theta\} \pi(\theta).
\]
The following two-step method is used to obtain the posterior draws of $\theta$

**Step 1.** Generate $\eta^*$ from a normal distribution with mean 0 and variance-covariance matrix $\hat{V}\{\hat{U}(\theta)\}$.

**Step 2.** Given $\eta^*$ generated from Step 1, obtain $\theta^*$ by solving $\hat{U}(\theta) = \eta^*$ for $\theta$.

If the solution of $\hat{U}(\theta)$ is not unique, we can generate $\theta^*$ from its posterior distribution directly.
Asymptotic result

Theorem

Under the regularity conditions, conditional on the full sample data,

\[ \left| \mathbb{P}\{ \theta \mid \hat{U}(\theta) \} - \phi_{\hat{\theta}, \hat{V}(\hat{\theta})}(\theta) \right| \to 0 \quad (9) \]

as \( n \to \infty \) almost surely, where \( \phi_{\mu, \Sigma}(x) \) is the probability density function of the normal distribution with mean \( \mu \) and variance-covariance matrix \( \Sigma \), \( \hat{\theta} \) is a consistent estimator obtained by solving \( \hat{U}(\theta) = 0 \), and \( \hat{V}(\hat{\theta}) \) is a consistent variance estimator of \( \text{var}(\hat{\theta}) \).

Note that the Bayesian inference is calibrated to the corresponding frequentist inference asymptotically.
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Estimating the population mean under informative sampling

- A probability sample consists of \( n \) elements of \((x_i, y_i)\) from a finite population.
- Let \( \theta = (\bar{X}, \bar{Y}) \), where \( \bar{X} = N^{-1} \sum_{i=1}^{N} x_i \) and \( \bar{Y} = N^{-1} \sum_{i=1}^{N} y_i \). In particular, we are interested in estimating \( \bar{Y} \).
- Let \( \hat{\theta} = (\hat{X}, \hat{Y}) \) be the design estimator of \( \theta \) and

\[
V = \begin{pmatrix}
V_{xx} & V_{xy} \\
V_{xy}^T & V_{yy}
\end{pmatrix}
\]

be the corresponding design variance estimator.
Let the prior distribution $\pi(\theta)$ be a normal distribution with mean $\mu = (\mu_x, \mu_y)$ and variance-covariance matrix
\[
\Lambda = \begin{pmatrix}
\Lambda_{xx} & \Lambda_{xy} \\
\Lambda_{yx} & \Lambda_{yy}
\end{pmatrix}.
\]
We consider two cases.

**Case I.** The population mean $\bar{X}$ is unknown. By setting the flat prior for $\mu$, we have

$$\bar{Y} \mid \hat{\theta} \sim \mathcal{N}(\hat{\bar{Y}}, V_{yy}).$$

**Case II.** The population mean $\bar{X}$ is known. Set $\mu_x = \bar{X}$, $\Lambda_{xx} = 0$, $\Lambda_{xy} = 0$, the diagonal elements of $\Lambda_{yy}$ diverge to $\infty$, and other hyper parameters to be any finite value. Then,

$$\bar{Y} \mid \hat{\theta} \sim \mathcal{N}\{\hat{Y} - V_{xy} V_{xx}^{-1} (\hat{X} - \bar{X}), V_{yy} - V_{xy} V_{xx}^{-1} V_{xy}\}.$$
Estimating the population mean by combining information from two samples

- Consider a situation that sample A and sample B are selected independently from the same population. Only $x$ is observed in sample A, and $(x, y)$ is observed in sample B.
- We are interested in estimating $\bar{Y} = N^{-1} \sum_{i=1}^{N} y_i$. 
Estimating the population mean by combining information from two samples (Cont’d)

- For the sample A,
  - Denote $\hat{X}_A$ and $V_{xx,A}$ be the design consistent estimator of $\bar{X}$ and its variance.
  - By setting flat prior for $\bar{X}$, we have
    \[
    \bar{X} \mid \hat{X}_A \sim N(\hat{X}_A, V_{xx,A}).
    \]

- For the sample B,
  - Design consistent estimator $\hat{\theta}_B = (\hat{X}_B, \hat{Y}_B)$ and its variance estimator
    \[
    V = \begin{pmatrix}
    V_{xx,B} & V_{xy,B} \\
    V_{xy,B}^T & V_{yy,B}
    \end{pmatrix}.
    \]
  - Thus, $\hat{\theta}_B \mid \theta \sim N(\theta, V)$. 
By setting a normal distribution with mean $\hat{X}_A$ and variance $V_{xx,A}$ as the prior for $\bar{X}$ and a flat prior for $\theta_2$, we can have

$$\bar{Y} \mid \hat{X}_A, \hat{X}_B, \hat{Y}_B \sim \mathcal{N}(\mu_p, \Sigma_p),$$

where

$$\mu_p = \hat{Y}_B + V_{xy,B}^{-1} V_{xx,B}^{-1} (V_{xx,A}^{-1} + V_{xx,B}^{-1})^{-1} V_{xx,A}^{-1} (\hat{X}_A - \hat{X}_B),$$

$$\Sigma_p = \{I - V_{xy,B}^T V_{xx,B}^{-1} (V_{xx,yB}^{-1} + V_{xx,A}^{-1})^{-1} V_{xx,B}^{-1} V_{xy,B} V_{yy,xB}^{-1}\}^{-1} V_{yy,xB},$$

$$V_{xx,yB} = V_{xx,B} - V_{xy,B} V_{yy,B}^{-1} V_{xy,B}^T,$$

$$V_{yy,xB} = V_{yy,B} - V_{xy,B}^T V_{xx,B}^{-1} V_{xy,B}.$$
Calibration estimation

- Let \( \{(x_i, y_i) : i = 1, \ldots, n\} \) be the sample, and assume that \( X_N = \sum_{i=1}^{N} x_i \) is available.
- The goal is to estimate \( \theta = \sum_{i=1}^{N} y_i \) by incorporating the auxiliary information.
- The consistent estimator \((\hat{\theta}, \hat{\lambda})\) can be obtained by solving

\[
\hat{U}(\theta, \lambda) = \sum_{i=1}^{n} w_i(\lambda) (x_i, y_i) - (X_N, \theta) = 0,
\]

where \( w_i(\lambda) = g_i(\lambda)/\pi_i \), and \( g_i(\lambda) \) is the weight adjustment factor satisfying \( g_i(0) = 1 \).
We can make inference about $\theta$ based on posterior draws of $\theta^*$ which can be generated by the following two-step procedure.

**Step 1.** Generate $\eta^*$ from normal distribution with mean 0 and variance-covariance matrix $\hat{V}\{\hat{U}(\theta, \lambda)\}$.

**Step 2.** Obtain $(\theta^*, \lambda^*)$ by solving $\hat{U}(\theta, \lambda) = \eta^*$, where $\eta^*$ is obtained from Step 1.
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Simulation setup

- Generate a finite population \( \{(x_i, y_i) : i = 1, \ldots, N\} \) of size \( N = 2,000 \) from the following model

\[
\begin{align*}
x_i &\sim N(\mu_x, \sigma_x^2), \\
y_i &\sim \text{Ber}(p_i),
\end{align*}
\]

where \( \text{logit}(p_i) = -4 + x_i \), and \( (\mu_x, \sigma_x) = (5, 1) \).

- Let \( \theta_1 = N^{-1} \sum_{i=1}^{N} x_i \), and \( \theta_2 = N^{-1} \sum_{i=1}^{N} y_i \).

- The goal is to estimate \( \theta_2 \).
Simulation setup (Cont’d)

- Probability proportional to size sampling is used to obtain the sample with size $n$, and we set $n = 100$ and $n = 500$.
- Let the size measure $z_i$ be generated from $z_i = x_i + y_i + e_i$ with $e_i \sim \text{Exp}(1)$.
- Two cases are considered.
  - Case I. Only the sample observations $(x_i, y_i)$, $i = 1, \ldots, n$, are available.
  - Case II. In addition to sample observations, the finite population mean $\theta_1 = \sum_{i=1}^{N} x_i / N$ is also available.
Three methods are compared.

1. The proposed semi-parametric Bayesian method.
   - The estimating equation is
     \[
     \hat{U}(\theta) = \sum_{i=1}^{n} \frac{1}{\pi_i} \{(x_i, y_i) - \theta\} = 0, \tag{10}
     \]
     where \(\theta = (\theta_1, \theta_2)\), and \(\pi_i = nz_i/(\sum_{k=1}^{N} z_k)\).
   - The posterior draw \(\theta^*\) is obtained by
     \[
     \theta^* \sim p\{\theta \mid \hat{U}(\theta)\} \propto g\{\hat{U}(\theta) \mid \theta\} \pi(\theta),
     \]
     where \(\hat{U}(\theta) \mid \theta \sim \mathcal{N}[0, \hat{V}\{\hat{U}(\theta)\}]\).
Parametric Bayesian method.

The density function is known as

\[ f(x, y; \eta) = f(x; \mu_x, \sigma_x)f(y \mid x; \beta_0, \beta_1), \]  

(11)

where \( \eta = (\mu_x, \sigma_x, \beta_0, \beta_1) \).

A flat prior for \( \eta \) is used for Case I, and a degenerated prior \( \pi(\eta) \propto I_{\{\theta_2\}}(\mu_x) \) is used for Case II, where \( I_{\{\theta_2\}}(\mu_x) = 1 \) if \( \mu_x = \theta_2 \), 0 otherwise.

Once \( \eta^* \) is obtained, we generate \((x_i^*, y_i^*)\) from \( f(x, y; \eta^*) \) and then compute \( \theta_2^* = N^{-1} \sum_{i=1}^{N} y_i^* \).
Bayesian penalized spline predictive method (Chen et al., 2010).

The suggested spline model is

$$\Phi^{-1}\{E(y_i; \alpha, b, \pi_i)\} = \alpha_0 + \alpha_1 \pi_i + \sum_{l=1}^{15} b_l (\pi_i - k_l)_+, \ i = 1, \ldots, n,$$

where $\Phi^{-1}(\cdot)$ is the inverse CDF of $N(0, 1)$, $b_l \sim N(0, \tau^2)$, $(x)_+ = x$ if $x \geq 0$, 0 otherwise. The fixed knots $k_l$ are chosen such that $m_\pi < k_1 < \cdots < k_{15} < M_\pi$ are equally spaced, where $m_\pi$ and $M_\pi$ are the minimum and maximum values among $\{\pi_i; i = 1, \ldots, n\}$.

Flat priors for $\alpha$ and $\tau^2$ are used.
Simulation results

Table: Monte Carlo simulation results, including Monte Carlo bias (Bias), standard error (S.E.) and coverage rate (C.R.) using the three methods: semiparametric Bayesian method (S.B.M.), parametric Bayesian method (P.B.M.), and the Bayesian penalized spline predictive method (B.P.S.P.)

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Method</th>
<th>Bias</th>
<th>Case I S.E. (×100)</th>
<th>Case I C.R. (%)</th>
<th>Case II S.E. (×100)</th>
<th>Case II C.R. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 100</td>
<td>S.B.M.</td>
<td>0.00</td>
<td>5.18</td>
<td>93.1</td>
<td>0.00</td>
<td>4.83</td>
</tr>
<tr>
<td></td>
<td>P.B.M.</td>
<td>0.00</td>
<td>4.93</td>
<td>94.2</td>
<td>0.00</td>
<td>4.51</td>
</tr>
<tr>
<td></td>
<td>B.P.S.P.</td>
<td>0.00</td>
<td>4.16</td>
<td>92.5</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>n = 500</td>
<td>S.B.M.</td>
<td>0.00</td>
<td>2.22</td>
<td>95.2</td>
<td>0.00</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>P.B.M.</td>
<td>0.00</td>
<td>2.26</td>
<td>95.1</td>
<td>0.00</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>B.P.S.P.</td>
<td>0.00</td>
<td>1.71</td>
<td>94.0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
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The proposed method is applied to analyze the Korean Longitudinal Study of Aging (KLoSA) sample of 2006.

A primary sample \((n_1 = 9,842)\) and a validation sample \((n_2 = 505)\) are provided, and the validation sample is a random sample from the primary sample.

The hypertension status \((Y)\), age \((X_2)\) and body mass index \((Z)\) through self-reported surveys are available in both samples, but the true body mass index \((X_1)\) is only available in the validation sample.

However, the body mass index in the primary sample \((Z)\) is subject to measurement errors.

The goal is to investigate the relationship between hypertension \((Y)\) and two factors, that is, body mass index \((X_1)\) and age \((X_2)\).
Background (Cont’d)

Figure: Relationship between the true body mass index ($X_1$) and the error-prone surrogate ($Z$) with the solid line showing $X_1 = Z$. 
Model setup

- We are interested in estimating parameters in the following logistic model for $Y$,

$$\text{logit}\{\text{pr}(Y = 1 \mid X_1, X_2)\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2.$$ 

- Since we do not observe $X_1$ in the primary sample, we can use its surrogate $Z$ by incorporating the following Berkson error model (Berkson, 1950)

$$X_1 = \alpha_0 + \alpha_1 Z + e, \quad (12)$$

where $e \sim N(0, \sigma^2)$.

- By the surrogate assumption, we have $f(y \mid x_1, x_2, z) = f(y \mid x_1, x_2)$ and so

$$f(x_1 \mid y, x_2, z) \propto f_1(y \mid x_1, x_2; \theta_1) f_2(x_1 \mid z; \theta_2)$$

holds, where $\theta_1 = (\beta_0, \beta_1, \beta_2)$ and $\theta_2 = (\alpha_0, \alpha_1, \sigma^2)$. 
Sampling procedure

- Flat prior is used for $\theta = (\theta_1, \theta_2)$.
- The following two-step procedure is used for implementation.

**Step 1.** Given $\theta^{(m)} = (\theta_1^{(m)}, \theta_2^{(m)})$, $x_1^{(m)}$ for the primary sample is obtained by

$$f(x_1 \mid y_i, x_2i, z_i; \theta^{(m)}) \propto f_1(y_i \mid x_1, x_2i; \theta_1^{(m)})f_2(x_1 \mid z_i; \theta_2^{(m)})$$
Sampling procedure (Cont’d)

Step 2. Based on $x_{1i}^{(m)}, i \in V^c$, $\theta_1^{(m+1)}$ can be generated by using

$$\sum_{i \in V} w_i S_1(\theta_1; y_i, x_{1i}, x_{2i}) + \sum_{i \in V^c} w_i S_1(\theta_1; y_i, x_{1i}^{(m)}, x_{2i}) = 0,$$

where $S_1(\theta_1; y, x_1, x_2) = \partial \log f_1(y \mid x_1, x_2; \theta_1)/\partial \theta_1$, and $V$ is the set of validate sample.

Similarly, we can generate $\theta_2^{(m+1)}$ by using

$$\sum_{i \in V} w_i S_2(\theta_2; x_{1i}, z_i) = 0,$$

where $S_2(\theta_2; x_1, z) = \partial \log f_2(x_1 \mid z; \theta_2)/\partial \theta_2$. 
Results

Table: Summary of data analysis using the proposed semiparametric Bayesian method and the fractional imputation (Kim et al., 2016)

<table>
<thead>
<tr>
<th>Method</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiparametric Bayes</td>
<td>Mean -9.7673</td>
<td>0.1945</td>
<td>0.0633</td>
</tr>
<tr>
<td></td>
<td>S.E. 0.3672</td>
<td>0.0114</td>
<td>0.0025</td>
</tr>
<tr>
<td>Fractional imputation</td>
<td>Mean -9.7487</td>
<td>0.1940</td>
<td>0.0633</td>
</tr>
<tr>
<td></td>
<td>S.E. 0.4128</td>
<td>0.0135</td>
<td>0.0023</td>
</tr>
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Conclusion

- A unified Bayesian method is proposed for the informative sampling case, and design features are incorporated in the estimating function.
- The inference from the proposed method is calibrated to the frequentist one automatically.
- It is easy to be implemented in practice.
Thank you.
Selected reference


